The following numerical values were used in calculating the temperature distribution: $T_1 = 82°C$ ($\Theta_1 = 0.1879$); $T_2 = 20°C$ ($\Theta_2 = 0$); $R = 80$ mm; $R_0 = 8$ mm; $\rho_0 = 0.1$; Pd = 350. The eigenvalues λ_n and λ_{nk} were obtained from the solution of the characteristic equations by using the McMahon formulas, given in [5]. Table 1 lists the roots of the present characteristic equations.

Figure 2a, b, c shows the distribution of the temperature Θ in a radial cross section at an angle φ for various Fo. It can be seen from the graphs that that for all practical purposes the steady state is reached for Fo $\geqslant 0.5$.

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FLOW OVER BLUNT BODIES WITH SPIKES AND CAVITIES

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The influence of the shape of bodies of revolution with complicated generating lines on the coefficient of drag is investigated by the method of "large particles."

It is known that even a slight change in the shape of the generating lines of bodies of revolution has a strong influence on the aerodynamic coefficient of drag [1, 2]. The introduction of new elements of the generating lines, such as the presence of special features of the cavern or spike type on the front surface, can have all the more pronounced an influence on c_x.

"Bow" separation zones are characteristic of the flows around such bodies. Ever more attention is presently being paid to the investigation of separation flows [3, 4, and others]. The conducting of experiments at high velocities is connected with considerable, at times fundamental, technical difficulties, and such natural experiments are very costly, too. Therefore, it is desirable to use a numerical experiment for the solution of such problems [5]. The method of "large particles" [6] is used in the present report. Its use is desirable because it allows one to study nonsteady flows during streamline flow over blunt bodies having generating lines of complicated shape (including bends) without the isolation of any singularities. The spectrum of velocities of the oncoming stream is sufficiently wide, including sub-, trans-, and supersonic modes. The bodies of revolution with generating lines of arbitrary configuration, including sections with bends and concavities, were calculated by the method of "fractional cells" [7].

An analysis of the experimental and numerical results obtained allows us to make the following basic classification of modes with streamline flow over bodies with spikes (Fig. 1). We note that nonsteady modes were not considered.

The pattern of streamline flow over a cylindrical body of revolution with a "short" spike, when the distance of withdrawal of the shock wave from the body over which the flow occurs is greater than the length of the spike, is shown in Fig. 1a.

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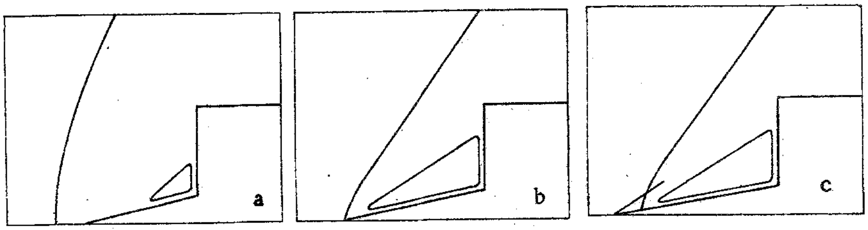

Fig. 1. Scheme of classification of flows around bodies with spikes.

The case of streamline flow over a body with a spike when the length of the spike is now sufficient to change the shape of the bow shock, converting it from almost straight (in the vicinity of the axis of symmetry) into an oblique shock, is shown schematically in Fig. 1b

The scheme of flow in the case when the length of the spike in front of the body is considerably greater than the distance of withdrawal of the shock wave is presented in Fig. 1c. Here the main compression shock develops from the stagnant zone ahead of the body. The modes of flow for bodies with cavities are classified similarly.

Let us dwell briefly on the statement of the problem. Euler's equations in a cylindrical coordinate system were taken as the initial equations. We considered two-dimensional streamline flow over blunt bodies of revolution by a gas stream with $\varkappa$ = const in a wide range of velocities of the oncoming stream ($0.9 \leqslant M_\infty \leqslant 7.0$). Spikes of different lengths, 0.4d, 0.6d, 0.8d, and 1.0d, were attached to the front parts of the blunt bodies. Spikes with cylindrica and conical shapes were taken. Special attention was paid to blunt spikes, since the sharp end of a spike melts and is blunted due to the development of high temperatures during motion with large hyper- and supersonic velocities. The numerical investigation was made on a BÉSM-6 computer by a program in the FORTRAN language. The region of integration comprised a field of 20 × 30 cells for bodies of revolution with cavities and a field of 40 × 58 cells for bodies with spikes. The time of calculation of one variant was 30-60 min. A steady-state solution was obtained as a result of establishment. We note that here one is able to investi gate not only purely supersonic and subsonic but also near-sonic flows, very important for practice from the point of view of flight stability and control, by a single algorithm. Some results of the calculations for supersonic modes of flow are presented below (at transonic velocities the effect of spikes is slight).

In the method of "large particles" one uses the separation of the initial system of differential equations by physical processes. For example, Euler's hydrodynamic equations are separated into three parts: the Eulerian, Lagrangian, and conclusive stages. When it is nece sary to allow for the effect of heat conduction one adds one more separated operator: the equation of heat conduction

$$\rho \frac{\partial I}{\partial t} = -\operatorname{div} \mathbf{q},$$

where I is the specific internal energy and $\mathbf{q}$ is the heat flux ($\mathbf{q} = -\chi(T)\operatorname{grad}T$). In this case all the other separated elements of the initial system of equations remain unchanged. Similarly, to allow for more complicated physicochemical effects one adds separated operators of radiation absorption and so forth to the algorithm. Then all the separated operators as a whole comprise the complete initial system of differential equations corresponding to the chosen physical model.

The introduction of heat conduction proved important in the study of the interaction of laser radiation with matter, for example. But in a number of gasdynamic problems the influence of heat conduction is small. Examples of the solution of problems from this class are presented below.

A numerical experiment conducted on the basis of the method of "large particles" to calculate the streamline flow over blunt bodies with spikes (a round cylinder with a flat end and a body of revolution with a curved generating line — an ellipsoid) made it possible to find the return-circulation zones ahead of them. We note that such flow patterns with the formation of front separation zones are observed in problems with jets directed opposite to the stream [8]. In this case the jet plays the role of an "effective spike." The coefficient c_x of the body can also be considerably reduced in this case. Separation of the stream

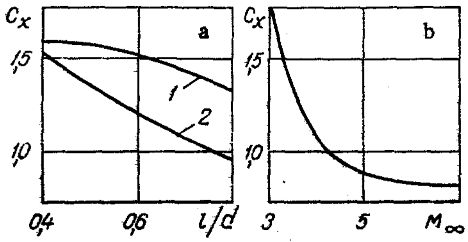

Fig. 2. Dependence of c_x: a) on length l of spike (1: M_∞ = 2.84; 2: 5.0); b) on Mach number M_∞.

occurs on sharp spikes. A withdrawn shock wave which is almost straight in the vicinity of the axis is converted into an oblique compression shock. It is important to correctly choose the parameters of the spike in order to achieve the maximum effect in the reduction of c_x. Since several schemes of streamline flow (Fig. 1a, b, c) are realized in practice, the generating line and the size of the spike are determined as a function of the Mach number M_∞ and the shape of the generating line of the body. By initiating the formation of a front separation zone spikes thereby promote an alteration in the shape of the effective surface of the body over which the external stream flows. The aerodynamic characteristics are altered in the process. Short spikes (whose length is less than the distance of withdrawal of the shock wave ahead of the body in the absence of a spike) have a weak influence on c_x [9]. If the length of the spike is close to the amount of withdrawal of the shock wave then one can detect a nonsteady mode of streamline flow, which can have a negative effect on the stability of the motion, especially in transonic modes. We note that a similar nonsteady mode has also been observed in problems with injection when the flow rate of the injected gas becomes less than some critical number [10]. In this case the bow compression shock is destroyed and converted into a system of oblique shock waves.

Continuing to increase the length of the spike further, we arrive at the case shown in Fig. 1c. Here the separation region reaches its maximum value and a further increase in the length of the spike does not lead to a significant decrease in c_x, since the effective generating line, consisting of part of the solid boundary and the contact surface separating the stagnant region from the external stream, remains practically unchanged. All the aerodynamic characteristics are determined by the contact-surface–body configuration. A weak compression shock "sits" on the spike extending from the effective body. It follows from structural considerations that it is undesirable to use a spike considerably longer than the stagnant zone ahead of the body.

The dependence of c_x on the length of the spike for a round cylinder with a flat end at different Mach numbers M_∞ is shown in Fig. 2a. We note that c_x was calculated from the well-known equation [11] which determines it through an integral of a function of the pressure over the generating line of the body. It is seen from the figure that the effect of the spike becomes significant with an increase in the velocity of the oncoming stream and permits a decrease in c_x by about 30%.

Figure 2b gives a concept of the nature of the dependence of c_x on the Mach number M_∞ for a round cylinder with a flat end. It is seen that c_x depends strongly on M_∞ at moderate supersonic velocities, while for $M_\infty \geqslant 5$ it emerges into an asymptotic form and becomes practically constant.

Let us consider gas flows around bodies of revolution with cavities on the front part. The problem of investigating the influence of cavities on the quantity c_x and on the streamline flow pattern as a whole is connected with the creation of bodies of revolution with the maximum wave resistance and is based on the fact that energy of the oncoming stream is additionally expended on vortex formation in the cavity. As the standard we take a round cylinder with a flat end. We calculate the influence of the sizes of the cavities on c_x by varying them both in depth and in height. The calculations were made at Mach numbers M_∞ = 0.9-5.0. Fractional cells were used in the calculations of flows around cavities with complicated configurations.

Let us briefly analyze the results obtained. The flow patterns constructed on the basis of the calculated data permit a clear determination of the location of the withdrawn shock waves. Turbulent nonsteady flows are often observed in the cavities themselves, i.e., the process inside them is essentially nonsteady. A steady mode of flow is also possible at other Mach numbers M_∞ not falling within the range under consideration [10]. For a given cavity height we calculated several variants of depths and investigated the influence of the latter on c_x. For a fixed Mach number M_∞ and h = 0.5d, c_x first increases somewhat (by ~2%) and then practically ceases to depend on the depth of the cavity (i.e., energy of the oncoming stream sufficed for the maintenance of circulation only in a certain volume).

With h = 0.5d and with change in the depth of the cavity from 0.1 to 0.5d the increase in c_x was ~3%. With a change in the height of the cavity from 0.5 to 0.9d (at a depth of 0.3d) the increase in c_x was ~2%. The maximum increase in c_x in comparison with that for the standard body was about 6% at the maximum possible height of the cavity and some optimum depth (the depth depends on the parameters of the oncoming stream: the higher the Mach number M_∞, the deeper the cavity should be). The effect of the cavity increases somewhat with an increase in M_∞.

For a larger increase in c_x it is desirable to make small openings in the walls of the cavity. A certain energy is expended in the drainage of gas through these channels, and some part of the energy of the oncoming stream goes to the recovery of the circulation zone in the cavity from which the gas partially leaks out through the openings. An analysis of the results showed that small openings increase c_x by 1-2% more. Analogous results were obtained for cavities with curved concave generating lines.

NOTATION

c_x, coefficient of drag; $\varkappa = c_p/c_v$; M_∞, Mach number in the undisturbed stream; d, middle diameter of model, m; $\bar{p} = p/p_\infty$, dimensionless pressure; h, height of cavity, m; l, length of spike, m.

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